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"I conceive it to be totally impossible that a complete and correct theory can leave such large and systematic discordances. And I express my opinion that there is still some serious defect in the lunar theory." (*Monthly Notices*, xxxiv., p. 5.)

The Astronomer Royal has tabulated these residual errors for groups of about 4½ years, between 1750 and 1851. It was thus easy to see how far the newly discovered term reduces or augments these residual discordances. It was therefore gratifying to find that for the period between 1780 and 1851 it completely accounts for them. For this period the mean of the outstanding positive errors is reduced to about 0″·7, and the mean of the outstanding negative errors is reduced to about 0″·6. It does not account so well for the residual discordances between 1765 and 1780, and fails to account for the errors in the period 1750–1765. I have, however, discovered a minor term which will partially account for the errors in these earlier observations between 1750 and 1770.

From the nature of the new term it is obvious that the value which has been deduced for it in the present paper is at best merely provisional. It will require most careful revision, and the higher powers of the disturbing forces must be taken into account. This will involve some labour, because, owing to the great eccentricity of the orbit of *Mars*, the convergency of the series which it involves is slow.. It is probable that when the complete value of the term is ascertained, it will be found to be slightly smaller than that depending on the first power of the disturbing forces. Under these conditions it would still better represent the observations. The existence of this term is so strongly supported by observation, from the manner in which it removes the discordances between theory and observation detected by Sir G. Airy, that I thought it better to publish it in its present form, rather than defer this for the considerable period which may elapse before I am able to verify these results.

I take this opportunity to announce that I have discovered other terms in the mean motion of the Moon, arising from the direct action of the planets, and having sensible coefficients. I hope to be able to bring these before the Society in a short period. One of these, having the comparatively short period of 60 years, appears to be strongly indicated by observation.

London, Nov. 2, 1877.

On Three Small Inequalities in the Mean Motion of the Earth, and an Inequality in the Mean Motion of Mars. By E. Neison, Esq.

Whilst engaged on some investigations connected with the perturbations in the motion of the Moon produced by the action of the planets, I came across several terms of long period in the

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mean motion of the Earth which it appeared might possibly have a sensible value. These terms depend on the second power of the disturbing force. Hitherto the only two terms of long period of this character which have been made known are the two terms discovered by Leverrier, and embodied in his solar tables. (*Annales de l'Observatoire*, tome iv.) These terms are, adopting Leverrier's notation,

$$6''\cdot410 \sin\{[4n''+3n^{\text{iv}}-8n''']\,t+\text{A}\}, \qquad 0''\cdot292 \sin\{[7n''-3n'-4n''']\,t+\text{A}_1\},$$

the first having a period of 1,800 years and the second a period of 300 years.

The first new term depends on three times the mean motion of the Earth, plus three times the mean motion of *Jupiter*, minus twice the mean motion of *Venus*. In fact $[3n''-2n'+3n^{\text{iv}}]=2419''$ only, so that the term has a period of 530 years, which might give it a sensible value, although multiplied by the fourth power of the eccentricity of *Jupiter*. On computing its value, it was found, however, that all the principal terms destroy each other, so that the largest coefficient does not amount to $0''\cdot01$. There is another term of still longer period, namely that depending on the relation $[6n''-5n'+4n''']=1140''$, but this also will be probably entirely insensible.

The argument of the second term depends on the mean motion of the Earth, plus twice the mean motion of *Saturn*, minus twice the mean motion of *Mars*. As $[n''+2n^{\text{v}}-2n''']=5868''$, and the term is of the first order of the eccentricities, it might have yielded a term with a sensible coefficient. On calculating the different terms, the only sensible one was found to be

$$+0''\cdot0178 \sin\{(n''-2n'''+2n^{\text{v}})\,t+\epsilon''-2\epsilon'''+2\epsilon^{\text{v}}+\varpi'''\}.$$

This term, although larger than many retained by Leverrier in his tables of the Sun, is insignificant. A similar but larger term must exist in the mean motion of *Mars*, and by calculating its value it was found to be

$$-0''\cdot1275 \sin\{(2n'''-n''-2n^{\text{v}})\,t+2\epsilon'''-\epsilon''-2\epsilon^{\text{v}}+\varpi''\}.$$

It has a period of 220·9 years, and, being capable of altering the geocentric longitude of *Mars* by nearly half a second of arc, deserves some attention.

The third term depends on ten times the mean motion of the Earth, minus six times the mean motion of *Venus*, and three times the mean motion of *Jupiter*, and is therefore analogous to the very important term discovered by Leverrier, and depending on the mean motion of the Earth, *Mars*, and *Jupiter*. Inasmuch as $[10n''-6n'-3n^{\text{iv}}]=-7846''$, the term has a period of 165 years. The principal terms mutually destroy each other, so that the coefficient is not one-tenth of what might have been expected.

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The main outstanding terms are the following:—

$$\begin{array}{l}
\quad\;\; '' \qquad\quad\;\; '' \qquad\qquad\quad '' \\
+\{0{\cdot}0811-0{\cdot}1578=-0{\cdot}0767\}\sin\{(10n''-6n'-3n^{\text{IV}})\,t+10\epsilon''-6\epsilon'-3\epsilon^{\text{IV}}-\varpi''\} \\
+\{0{\cdot}0273-0{\cdot}0384=-0{\cdot}0111\}\sin\{(10n''-6n'-3n^{\text{IV}})\,t+10\epsilon''-6\epsilon'-3\epsilon^{\text{IV}}-\varpi'\} \\
+\{0{\cdot}0080-0{\cdot}0171=-0{\cdot}0091\}\sin\{(10n''-6n'-3n^{\text{IV}})\,t+10\epsilon''-6\epsilon'-3\epsilon^{\text{IV}}+\varpi^{\text{IV}}-2\varpi''\}. \\
+\qquad\qquad\qquad\qquad\{-0{\cdot}0057\}\sin\{(10n''-6n'-3n^{\text{IV}})\,t+10\epsilon''-6\epsilon'-3\epsilon^{\text{IV}}+\varpi^{\text{IV}}-\varpi''-\varpi'\}.
\end{array}$$

There is a peculiarity about this term which I am in hopes of examining. According to Leverrier the mean motion of *Neptune* is equal to $7872''{\cdot}7$. Therefore the term just considered in the mean motion of the Earth will give rise to another term of the form

$$\{10n''-6n'-3n^{\text{IV}}-n^{\text{VII}}\},=-26''.$$

This will be a term independent of the eccentricities, and of the third power of the disturbing forces, and with an immensely long period of 50,000 years, or nearly twice the period of the revolution of the Earth's perihelion. It is therefore very probable that this term possesses a comparatively considerable coefficient. It is true, from its immense length of period, the term is without importance as far as tables of the position of the Earth is concerned, but it possesses a scientific interest. A slight change in the value of the mean motion of *Neptune* would convert this term into a secular term, by making

$$\{[10n''+2\varpi''-6n'-3n^{\text{IV}}-n^{\text{VII}}]\}=0.$$

My principal reason for communicating these results is to let it be known that they have been examined. Hansen calculated many terms of this nature in the theory of the Earth, but he has nowhere stated what terms he so examined. This is to be regretted, as it leaves this fruitless and laborious work to be performed again by anyone traversing the same ground. Similarly, had Hansen and Delaunay published a list of the terms produced by the planets in the motion of the Moon which they had examined, it would have materially lightened the labour of anyone endeavouring to discover the missing terms of long period in the motion of the Moon.

Oct. 29, 1877.

The Satellite of Neptune. By J. W. Ward, Belfast.

The following observations were all made without any knowledge of the position angles or distances by Ephemeris, which was *afterwards* consulted. On the whole I do not think that this satellite is more difficult than the two outer satellites of *Uranus* as observed in the spring of 1876. The points were glimpsed by averted vision.